

Partial Dynamical Symmetries in the $f_{7/2}$ and $g_{9/2}$ Shells

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Abstract

Previous work on partial dynamical symmetries in the $f_{7/2}$ shell is extended to other shells e.g. $g_{9/2}$. The nuclei involved are ^{43}Sc , ^{43}Ti , ^{44}Ti , ^{52}Fe , ^{53}Fe , ^{53}Co , ^{96}Cd , ^{97}Cd and ^{97}In .

Previously we found partial dynamical symmetries (PDS) in the $f_{7/2}$ shell when, in a single j shell model calculation we set all $T=0$ two body matrix elements to zero [1,2,3,4,5]. We found that for selected angular momenta J_p and J_n were separately good quantum numbers-not just total J . Further states with a given (J_p, J_n) were all degenerate in energy for these selected states.

It took a while but it was finally realized that the selected states were those with angular momenta that could not occur for a system of identical particles. For example in ^{43}Ca there are no states in the $f_{7/2}$ model space with angular momenta $J=1/2$ and $J=13/2$. Therefore states in ^{43}Sc with these total J values are degenerate and have as good quantum numbers $(J_p=4, J_n=7/2)$. A similar story for $J=17/2$ and $19/2$. Here the good quantum numbers are $(J_p=6, J_n=7/2)$. In ^{44}Ca there are no states in the $f_{7/2}$ configuration with $J=3, 7, 9, 10, 11, 12$. States in ^{44}Ti and ^{52}Fe with $J_p=4, J_n=6$ and $J_p=6, J_n=4$ with angular momenta $J=3_2, 7_2, 9_1$ and 10_1 are all degenerate. Also $J_n=6, J_p=6, J=10_2$ and 12_1 . The $12+$ state is isomeric in ^{44}Ti but even more so ^{52}Fe . In the later nucleus the $12+$ state is at a lower energy than the $10+$ state. In ^{44}Ti it is a bit above.

We can carry over the arguments to the $g_{9/2}$ shell. Starting from a core with $Z=50, N=50$ the obvious analogs to nuclei in the $f_{7/2}$ shell are ^{97}Cd (^{97}In) and ^{96}Cd , three holes and four holes respectively. We stay away from particles added to an $N=40, Z=40$ core because ^{80}Zr is not a good closed shell.

In our previous papers we actually gave formulas for general j , not just $j=7/2$.

There are two conditions which lead to a PDS. The off diagonal condition insures that J_p and J_n are separately good quantum numbers-not just total J . Then there is the diagonal condition that explains why states with the same (J_p, J_n) are degenerate.

Three particles-off diagonal condition:

$$\left\{ \begin{array}{ccc} j & j & (2j-3) \\ (3j-4) & j & (2j-1) \end{array} \right\} = 0$$

Three particles diagonal condition:

$$\left\{ \begin{array}{ccc} j & j & (2j-1) \\ j & J & (2j-1) \end{array} \right\} = (-1)^{(J+j)}/(8j-2) \text{ for } J = (3j-1), (3j-2) \text{ and } (3j-4).$$

Four particles off-diagonal condition :

$$\left\{ \begin{array}{ccc} j & j & (2j-1) \\ j & j & (2j-1) \\ (2j-1) & (2j-3) & (4j-4) \end{array} \right\} = 0$$

Four particles diagonal condition:

$$\left\{ \begin{array}{ccc} j & j & (2j-3) \\ j & j & (2j-1) \\ (2j-3) & (2j-1) & J \end{array} \right\} = 1/[4(4j-5)(4j-1)] \text{ for selected } J \text{ values.}$$

This topic is also of interest in terms of what we call companion problems[6]. Initially Shadow Robinson and I used

Regge 6j symmetries to show why certain 6j symbols vanished [7]. But there are no Regge relations for 9j symbols. But then we found that Talmi [8] had shown for a completely different reason why the same 6j vanished. He constructed a coefficient of fractional parentage for a state with an angular momentum which did not exist for a system of three identical particles ($J=13/2$ in ^{43}Ca). The vanishing of the cfp was carried by the same 6j symbol we needed to explain the vanishing of off diagonal coupling for our PDS. We then used these ideas for 9j symbols. For a 4 nucleon system we calculate cfp's for states that do not exist.

In another direction with regards to companion problems Zhao and Arima [9] obtained the same 9j relations that we had by considering J pairing Hamiltonians. It is quite fascinating that quite different physical problems lead to the same mathematical relations.

Let us look at proceed systematically. For three identical particles in a j shell the maximum J is $j + (j-1) + (j-2) = (3j-3)$. For one proton and 2 neutrons the maximum value is $(2j-1) + j = (3j-1)$. Hence states with $J=3j-2$ and $3j-1$ are part of the PDS. These have high spins and so the single j model might work better. Also belonging to the PDS, are states with $J=1/2$ and $3j-4$. The last one belongs because there are no states with $J=J(\text{max}) - 1$ for identical fermions (also true for identical bosons).

For 4 nucleons (or holes) the maximum J is $j + (j-1) + (j-2) + (j-3) = 4j-6$. However for two protons and 2 neutrons the maximum J is $(2j-1) + (2j-1) = 4j-2$. Hence states with $J = (4j-5), (4j-4), (4j-3)$ and $(4j-2)$ belong to the PDS. These are high spin states. The single j shell might work fairly well for these. There might be other states with PDS, e.g. as noted above, $J=3$ and 7 in the $f_{7/2}$ shell and $J=11$ in the $g_{9/2}$ shell.

Consider first 3 nucleons in the $g_{9/2}$ shell. If they are identical $J_{\text{max}} = 21/2$. For one proton and 2 neutrons and/or

2 protons and one neutron $J_{\text{max}} = 25/2$. We get a degenerate set $J_p = 8$ $J_n = 9/2$ $J = 19/2, 23/2$ and $25/2$ (all $T=1/2$).

Consider four nucleons in the $g_{9/2}$ shell. If they are identical $J_{max} = 12$. For 2 protons and 2 neutrons $J_{max} = 16$.

Here are selected sets of degenerate states for four nucleons in the $g_{9/2}$ shell.

J_p	J_n	
8	8	$J=14, 16 \quad T=0$
8	6	$J=11, 13, 14 \quad T=0$

There are more. In the above (8,6) is an abbreviation for $(8,6) + (-1)^{(J+T)}(6,8)$. (For the (8,6) configuration there is also a degeneracy of $J=8$ and $J=9$. The above considerations do not explain this.)

The $T=0$ calculation is a good starting point to see the effects of putting back the $T=0$ matrix elements. In the $f_{7/2}$ shell. There was some striking behavior for $T=1/2$ states of a three particle system. In a complete fp calculation we considered the difference $E(\text{full}) - E(T=0=0)$. The behaviour for $J < 7/2$ was different from that for $J > 7/2$. In the former case for decreasing J the above quantity became increasingly and linearly negatives. For the higher spins these was as staggering effect with the alternate spins $J=9/2, 13/2$ and $17/2$ going up in energy when both $T=0$ and $T=1$ matrix elements were included while $J=7/2, J=11/2, 15/2$ and $19/2$ hardly changed. We should expect similar behaviour in higher j shells. For $T=0$ states of a four nucleon system we found that the odd spin states were pushed up significantly more than even spin when the full interaction was reintroduced. Thus with only the $E(T=0=0)$ interaction there were several odd spin states close to or below the lowest $J=12+$ state. These were pushed up by the full interaction.

There have been several shell model calculations in the $g_{9/2}$ region including early calculations by Auerbach and

Talmi [10], Serduke, Lawson and Gloeckner [11] and Ogawa [12]. The phrase "spin gap isomers is used" and Ogawa

predicted many such isomers in this region $^{95}\text{Pd}, ^{95}\text{Ag}, ^{96}\text{Cd}, ^{97}\text{Cd}$. Reintroducing the $T=0$ two-body matrix elements clearly helps to create these spin gaps.

At this workshop (Nuclear Physics in Astrophysics V) new results on ^{96}Cd have been presented. In particular his York

group found a $J=16+$ isomeric state. For this state to be isomeric the $J=14+$ state should be at a higher energy than the

$J=16+$ state. As note above without the $T=0$ interaction in the single j shell calculation these two states would be

degenerate in energy. The $T=0$ interaction is required to remove the degeneracy and push the $J=14+$ state above the $J=16+$ state.

There are other PDS in a single j shell calculation for ^{96}Cd . In general seniority is not a good quantum number in the

$g_{9/2}$ shell. However we previously found [10] that for a system of four identical particles (holes) with configuration $(g_{9/2})^4$ there is a certain $J=4 \quad v=4$ (as $J=6$) state that does not mix with the other two states, the latter having seniorities $v=2$ and 4 . In other words there is one eigenfunction for $J=4 \quad v=4$ which emerges no matter what interaction is used. This problem has been studied in

several references [13],[14],[15], and [16]. Different, and very wide ranging topics involving partial dynamical symmetries have been developed and reviewed by A. Leviatan [17].

As seen in the references these works were done with several collaborators Shadow Robinson (Phd thesis), Alberto Escuderos , Ben Bayman and Piet Van Isacker[1,2,3,4,5,15]. They were strongly influenced by works of Igal Talmi[8].

This work was stimulated in part by reports at the Weizmann Institute workshop following the Eilat conference of relevant experiments in the $g_{9/2}$ shell by the York group [19]. As reported by B.S. Nara Singh A $J=16+$ isomer was found in ^{96}Cd which decayed to a $J=15+$ isomer in ^{96}Ag [19]. In the light of his comments I re-examined work I had done with Escuderos [13] where the main thrust was not isomerism but symmetries. But as a residual we did obtain an isomerism of $J=15+$ in ^{96}Ag in a single j-shell calculation - $g_{9/2}$ - with a Q.Q interaction. The energies of the $J=15+, 14+, 13+, 12+, 11+$ states relative to a $J=1+$ ground state are respectively 2.48, 3.09, 2.53, 2.59 and 1.96 MeV. Thus the $J=12+, 13+$, and $14+$ are higher in energy than $J=15+$. The latter state can only decay to $J=11+$ via an E(4) and/or M(5) transition. Amusingly we find that the $J= 8+$ state is also isomeric in this model space.

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